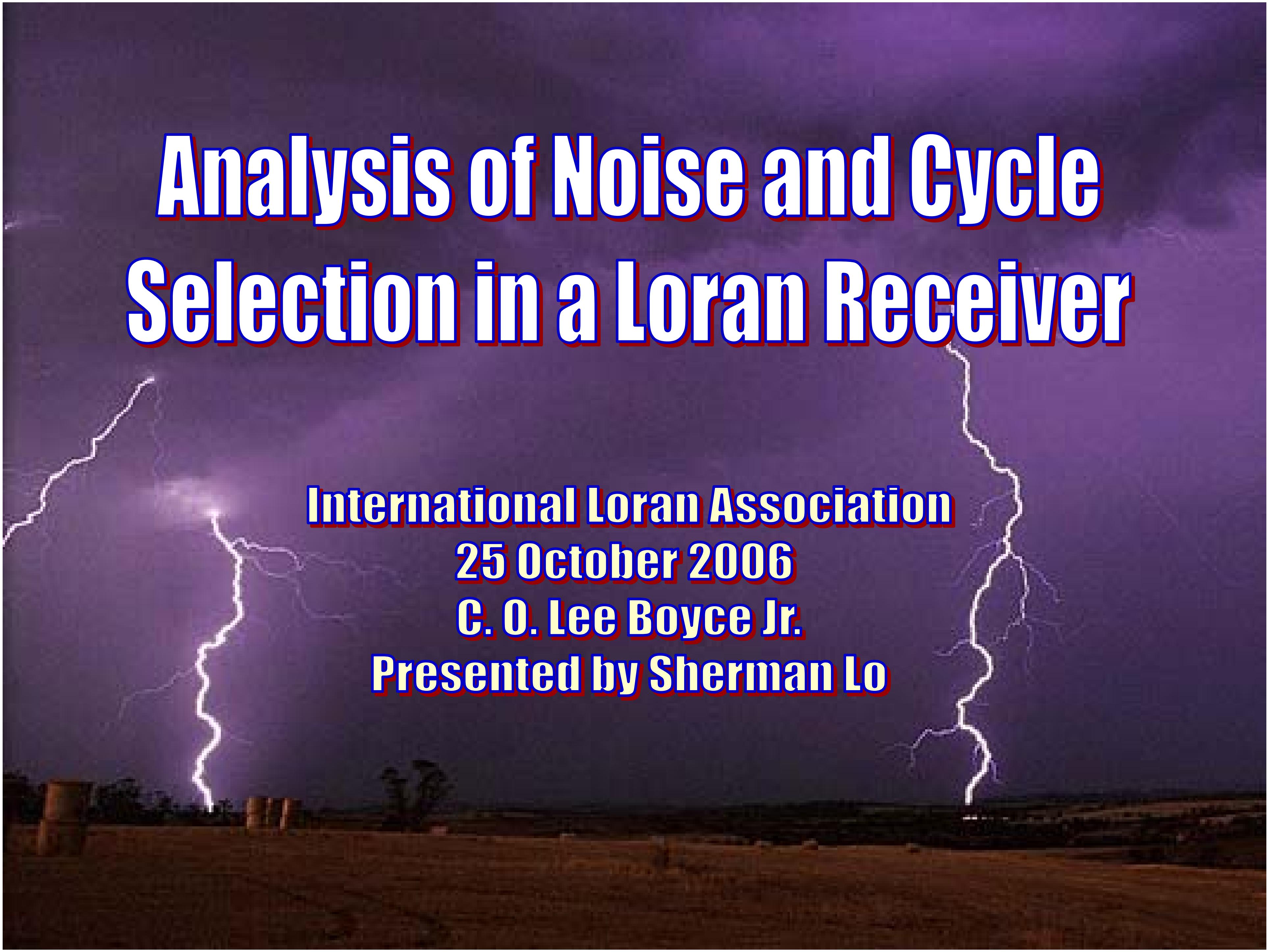

Analysis of Noise and Cycle Selection in a Loran Receiver

International Loran Association

25 October 2006

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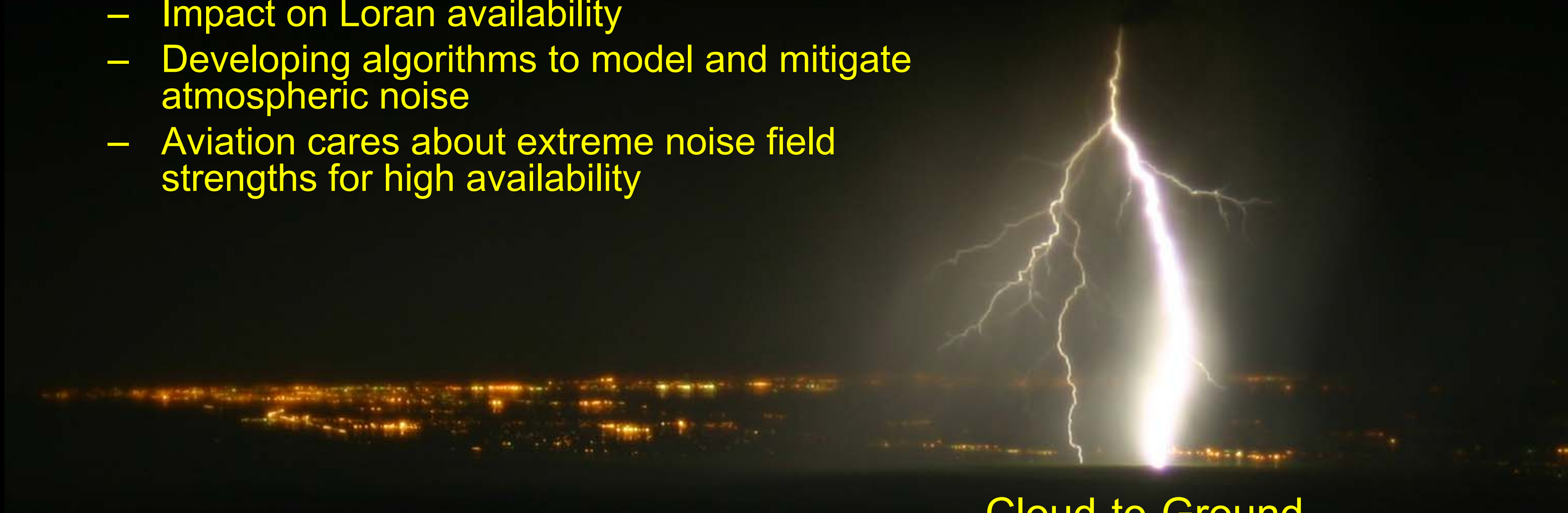
Presented by Sherman Lo

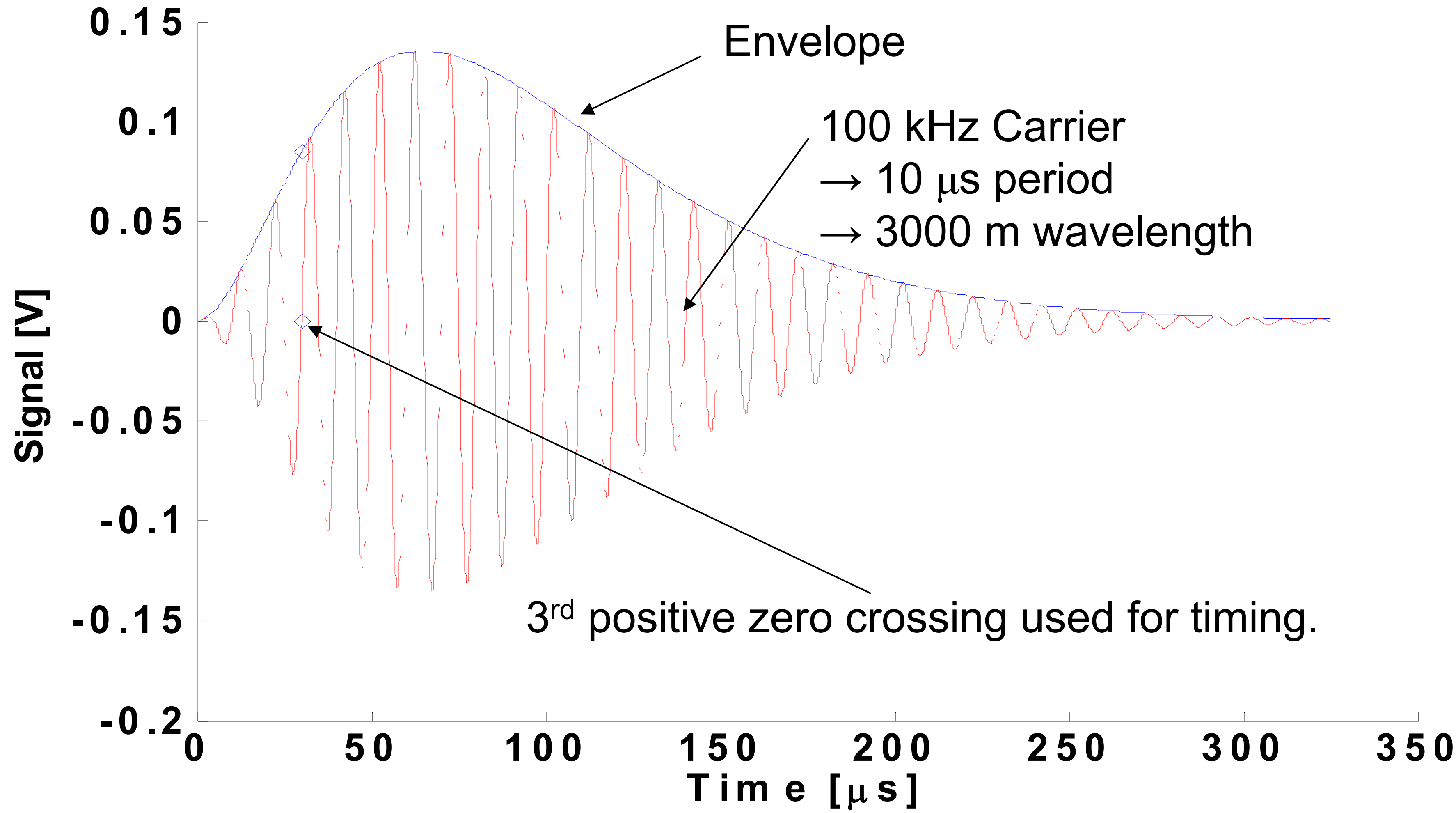
Atmospheric Noise

- Characteristics
  - Generated by atmospheric discharges (cloud-to-ground, cloud-to-cloud, intra-cloud)
  - Non-Gaussian
  - Correlated in time – “bursty”
  - Large amplitude variation
  - Requires more than mean and variance
  - CCIR defines the standard model
- Importance
  - Noise can cause a wrong cycle select
  - Impact on Loran availability
  - Developing algorithms to model and mitigate atmospheric noise
  - Aviation cares about extreme noise field strengths for high availability



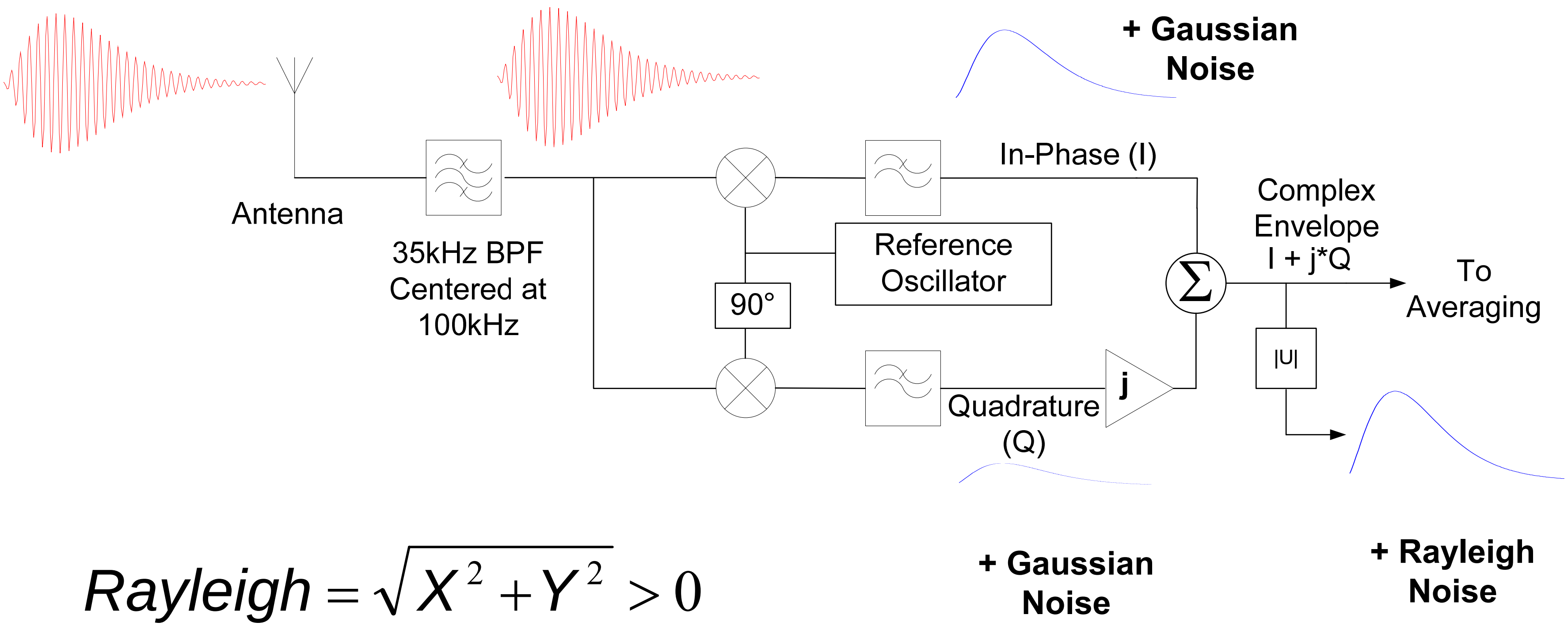
Cloud-to-Ground
Return Stroke

Loran Pulse



Envelope
100 kHz Carrier
→ 10 μs period
→ 3000 m wavelength
3rd positive zero crossing used for timing.
Signal [V]
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
50
100
150
200
250
300
350
Time [μs]

Loran Front End

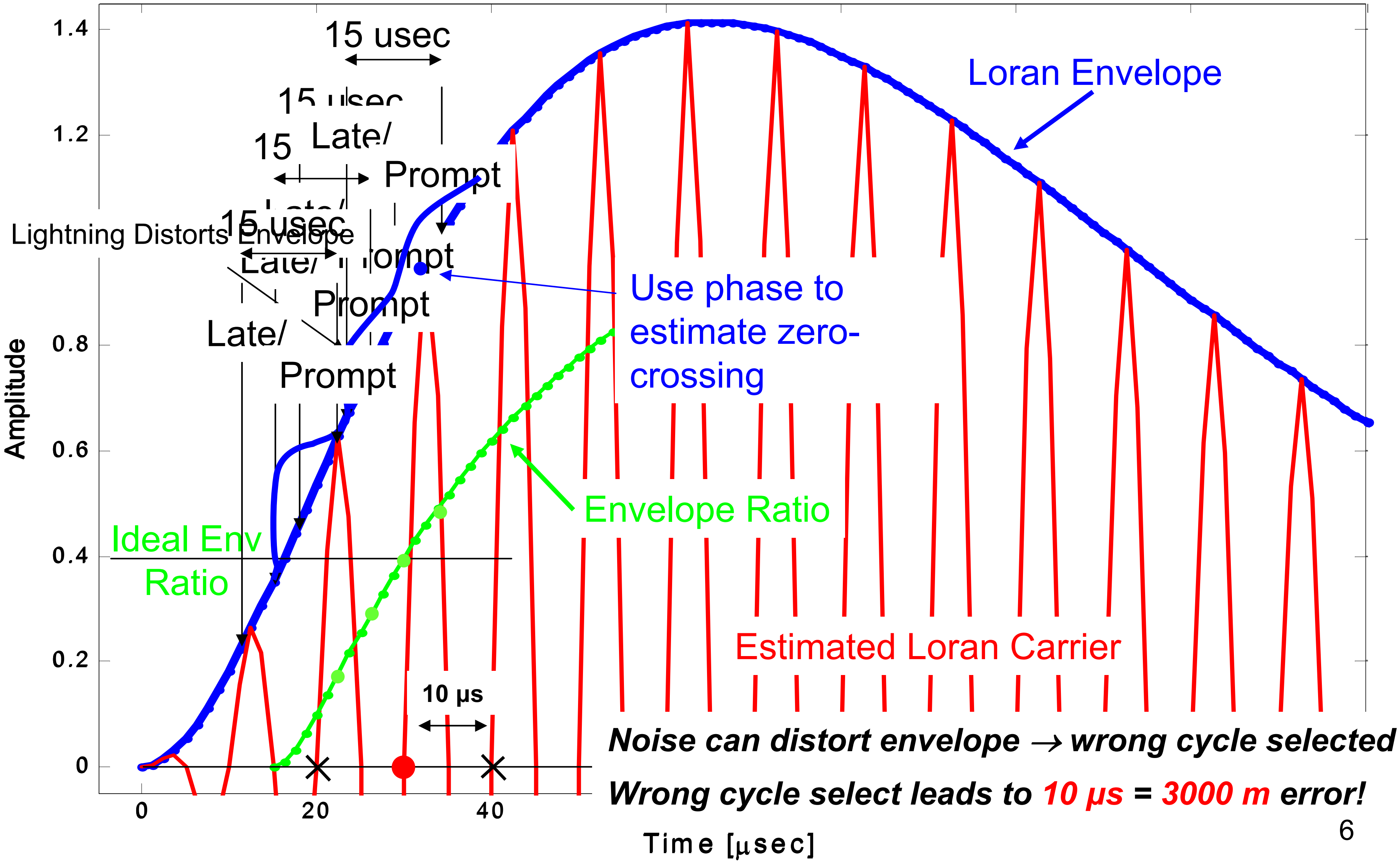


$$Rayleigh = \sqrt{X^2 + Y^2} > 0$$

$$X, Y \sim N(0,1)$$

Envelope Statistics

Ratio Test for Timing



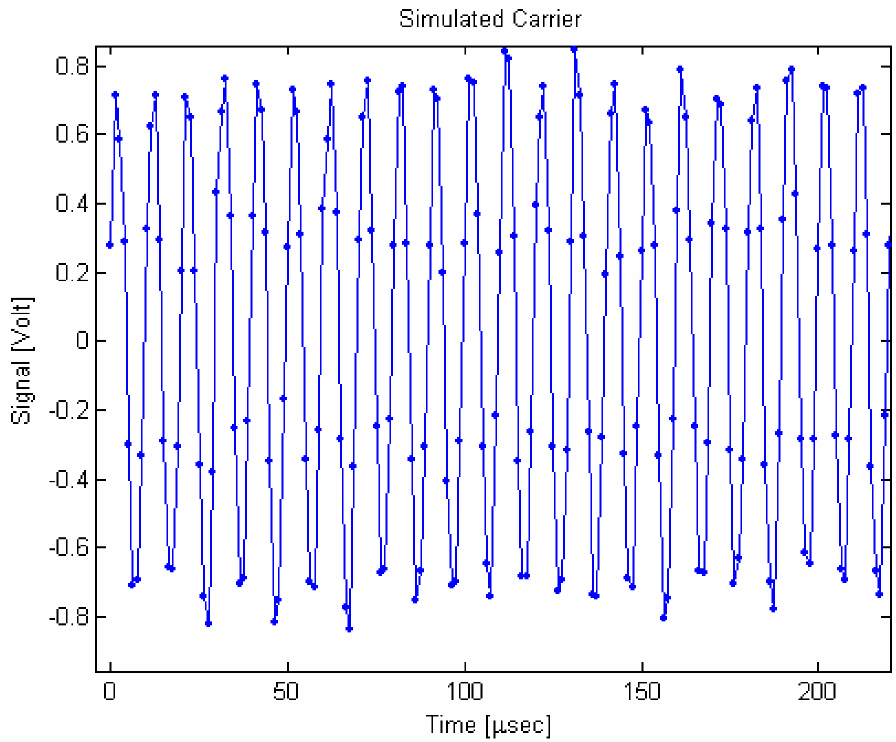
Noise can distort envelope → wrong cycle selected

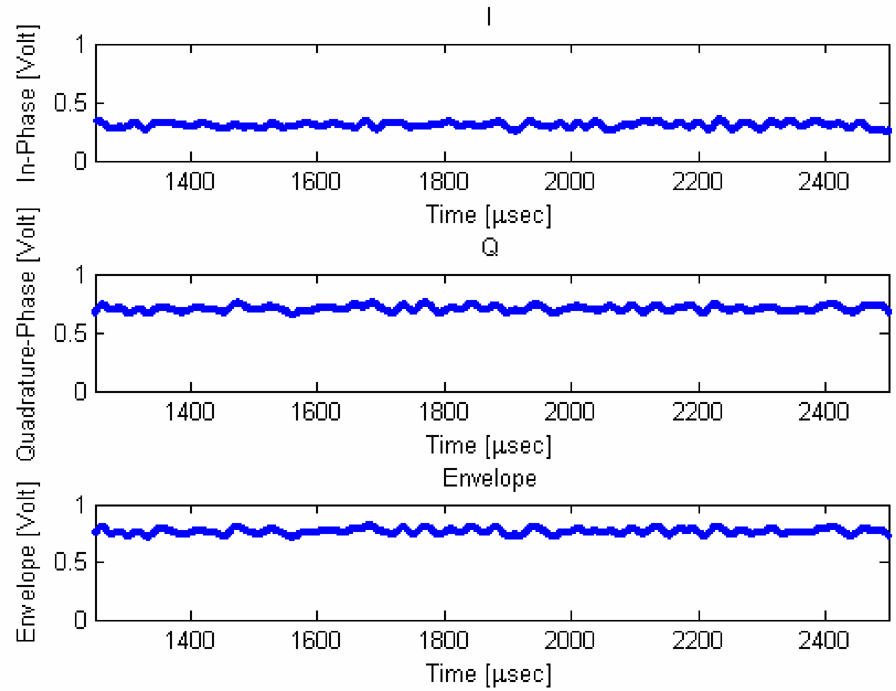
Wrong cycle select leads to 10 µs = 3000 m error!

Envelope Statistics

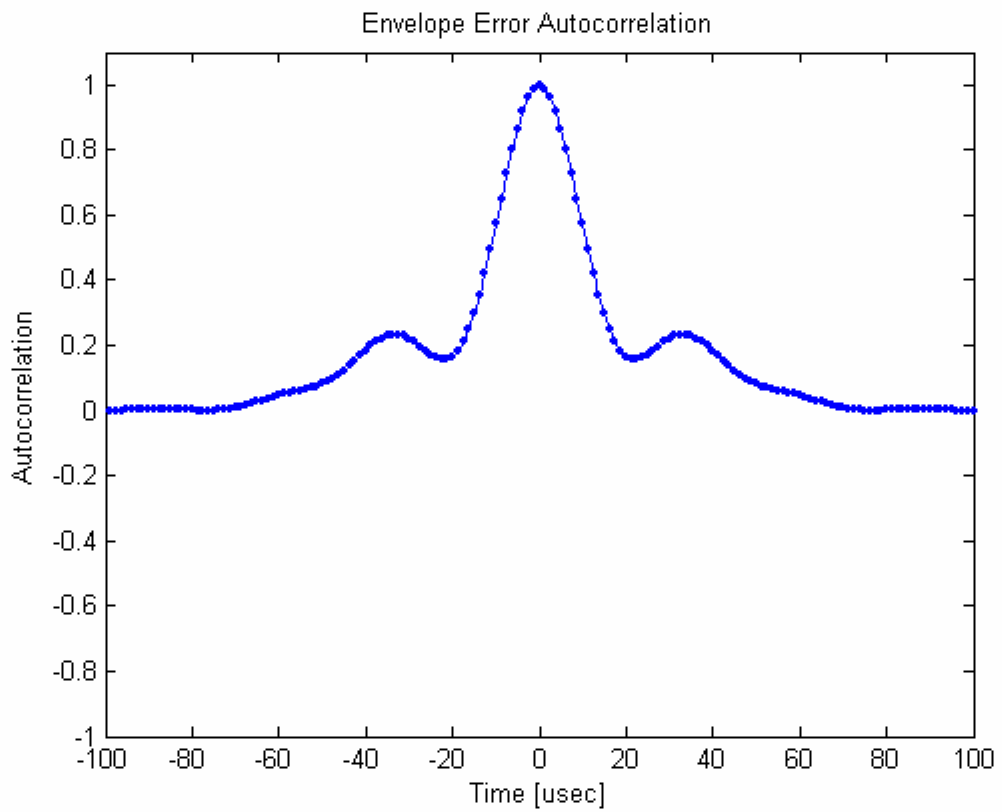
- Wanted to know what happens when Gaussian noise is added to a real signal.
- Took a 100kHz carrier and added Gaussian noise
- Verified that the noise on the I and Q channels are Gaussian and Independent
- Envelope is Rayleigh

Simulated Carrier & Mix

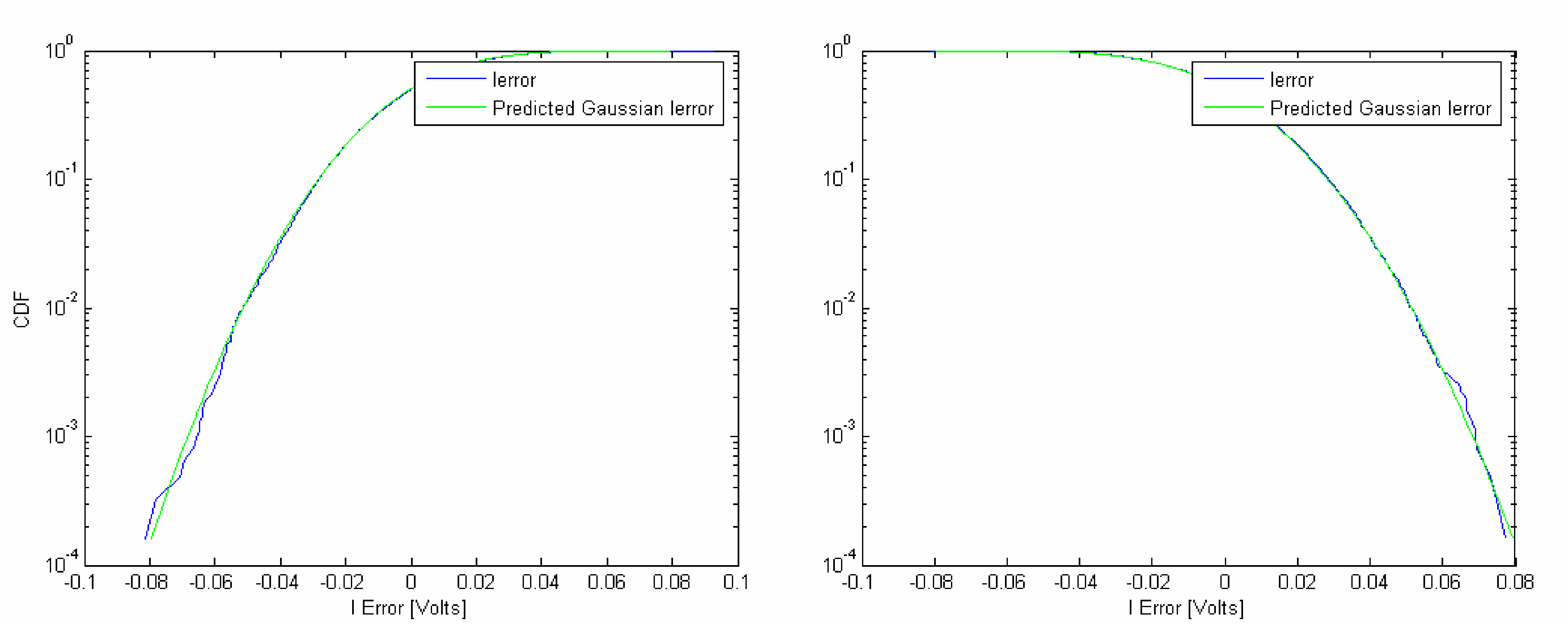




Autocorrelation



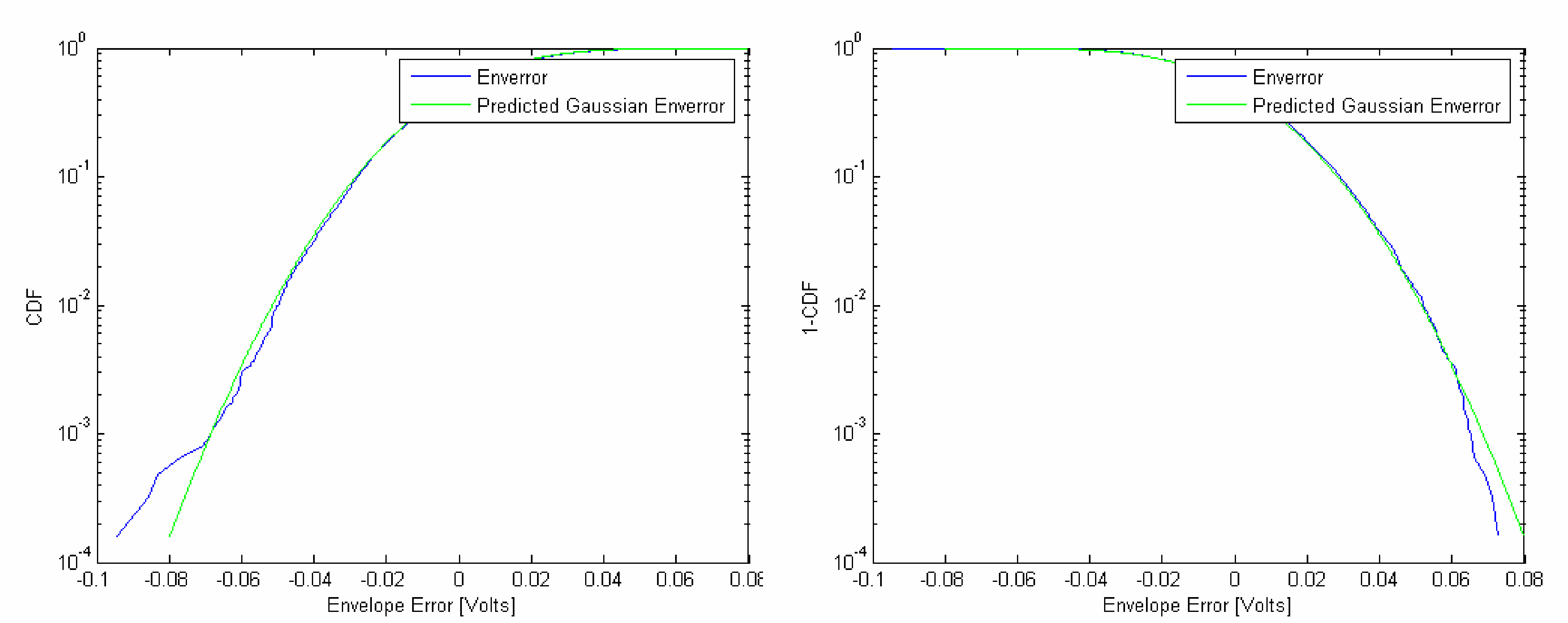
I & Q Errors are Gaussian



- Q is similar

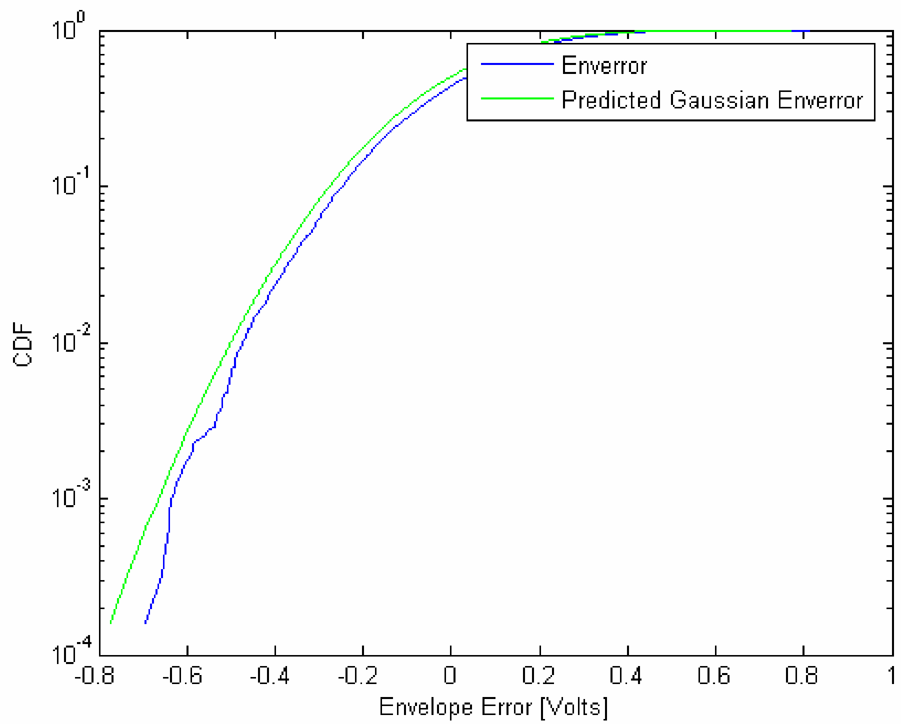
Envelope at different SNR

SNR = 20dB

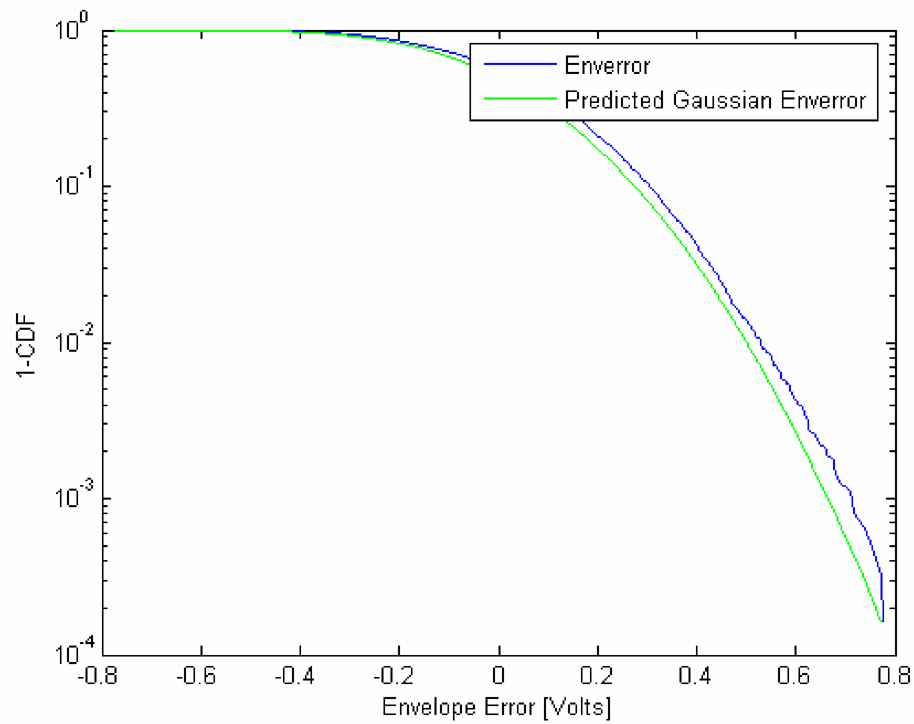


Enverror
Predicted Gaussian Enverror
CDF
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
-0.1 -0.08 -0.06 -0.04 -0.02 0 0.02 0.04 0.06 0.08
Envelope Error [Volts]
Enverror
Predicted Gaussian Enverror
1-CDF
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
-0.1 -0.08 -0.06 -0.04 -0.02 0 0.02 0.04 0.06 0.08
Envelope Error [Volts]

SNR 0dB

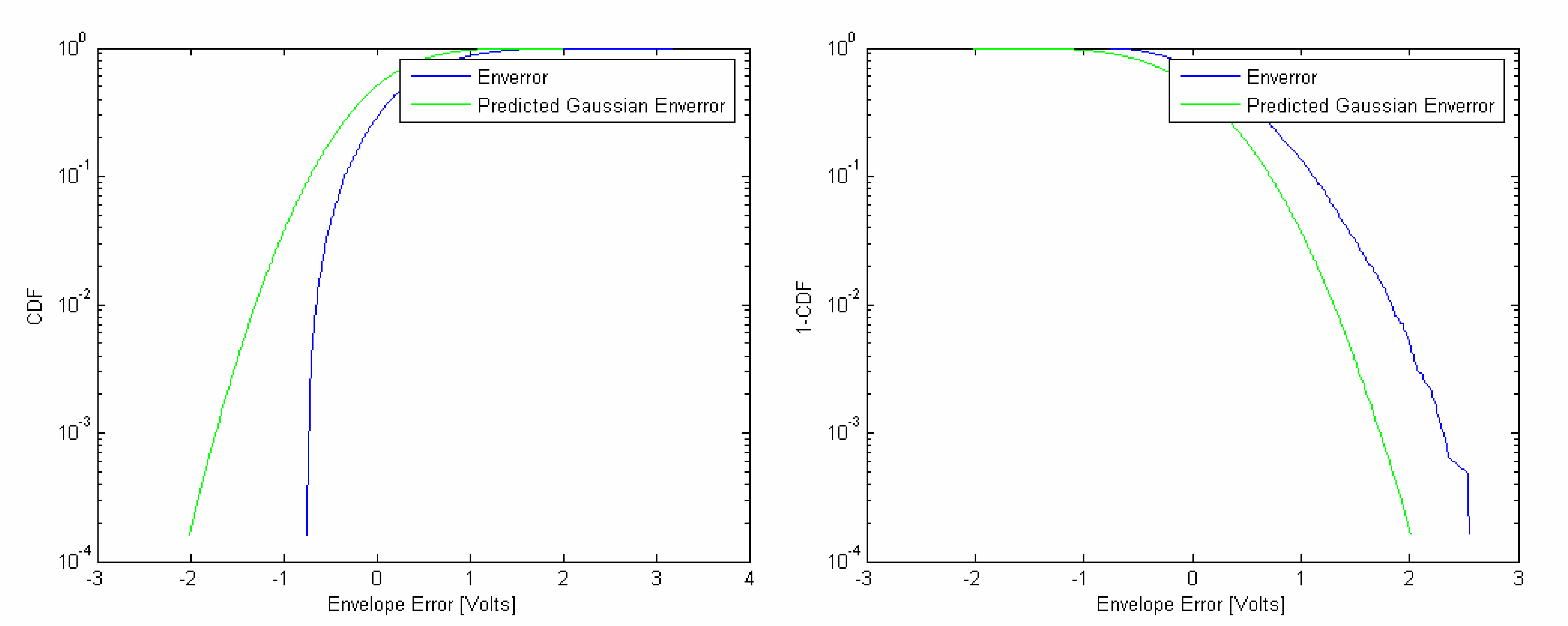


CDF
Enverror
Predicted Gaussian Enverror
Envelope Error [Volts]



1-CDF
Enverror
Predicted Gaussian Enverror
Envelope Error [Volts]

SNR -10dB



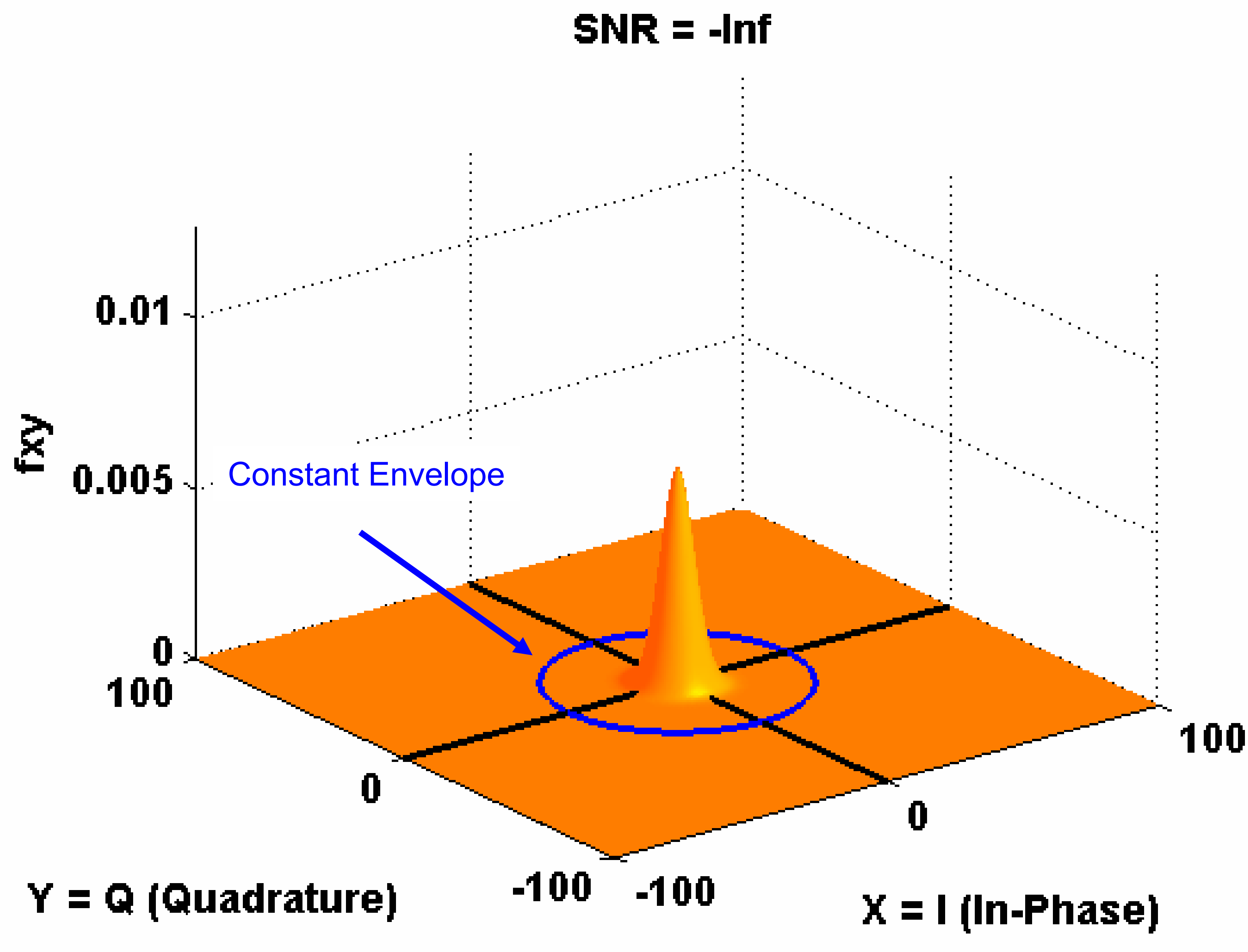
CDF
Enverror
Predicted Gaussian Enverror
Envelope Error [Volts]
1-CDF
Enverror
Predicted Gaussian Enverror
Envelope Error [Volts]

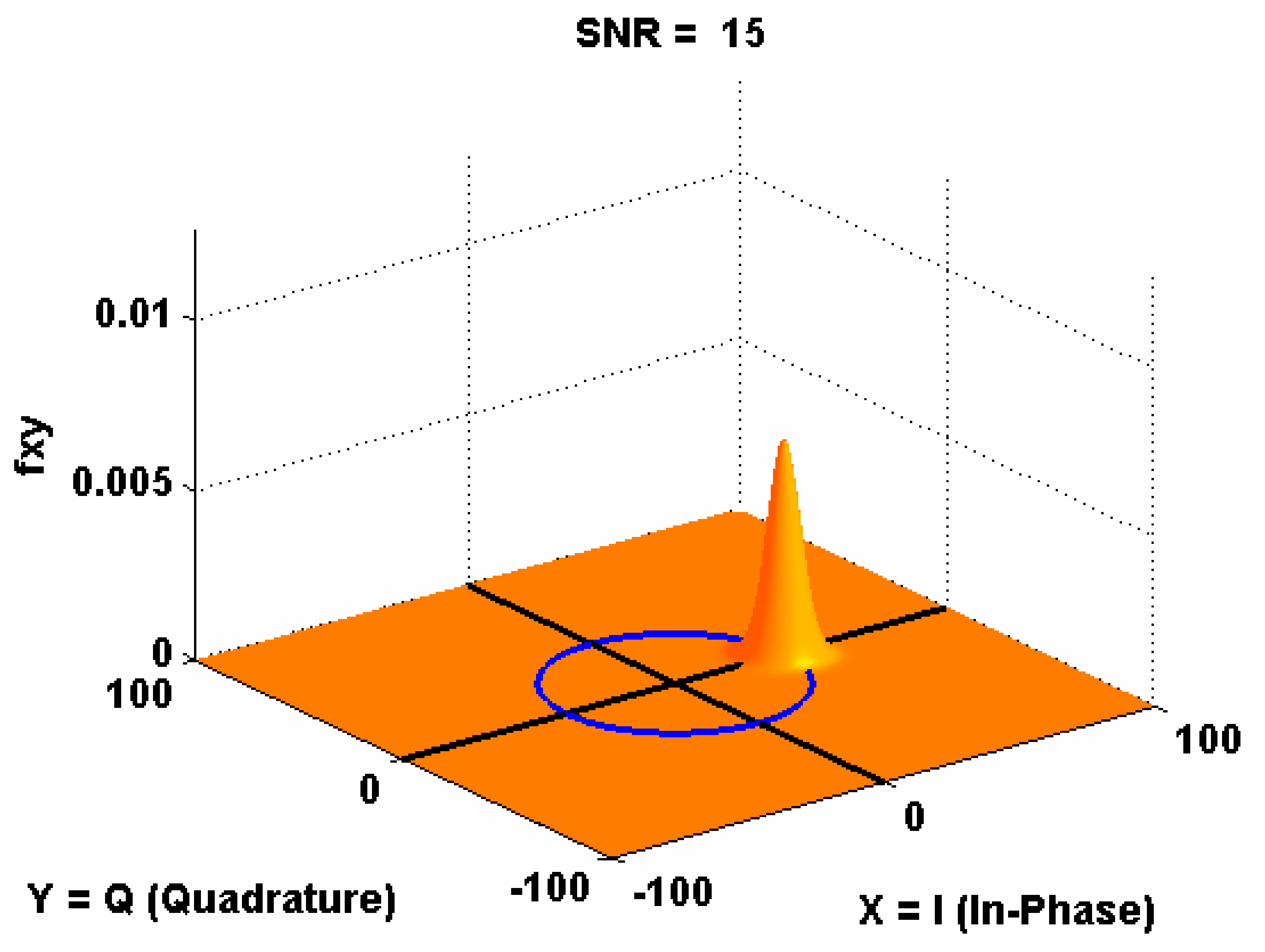
Exact CDF

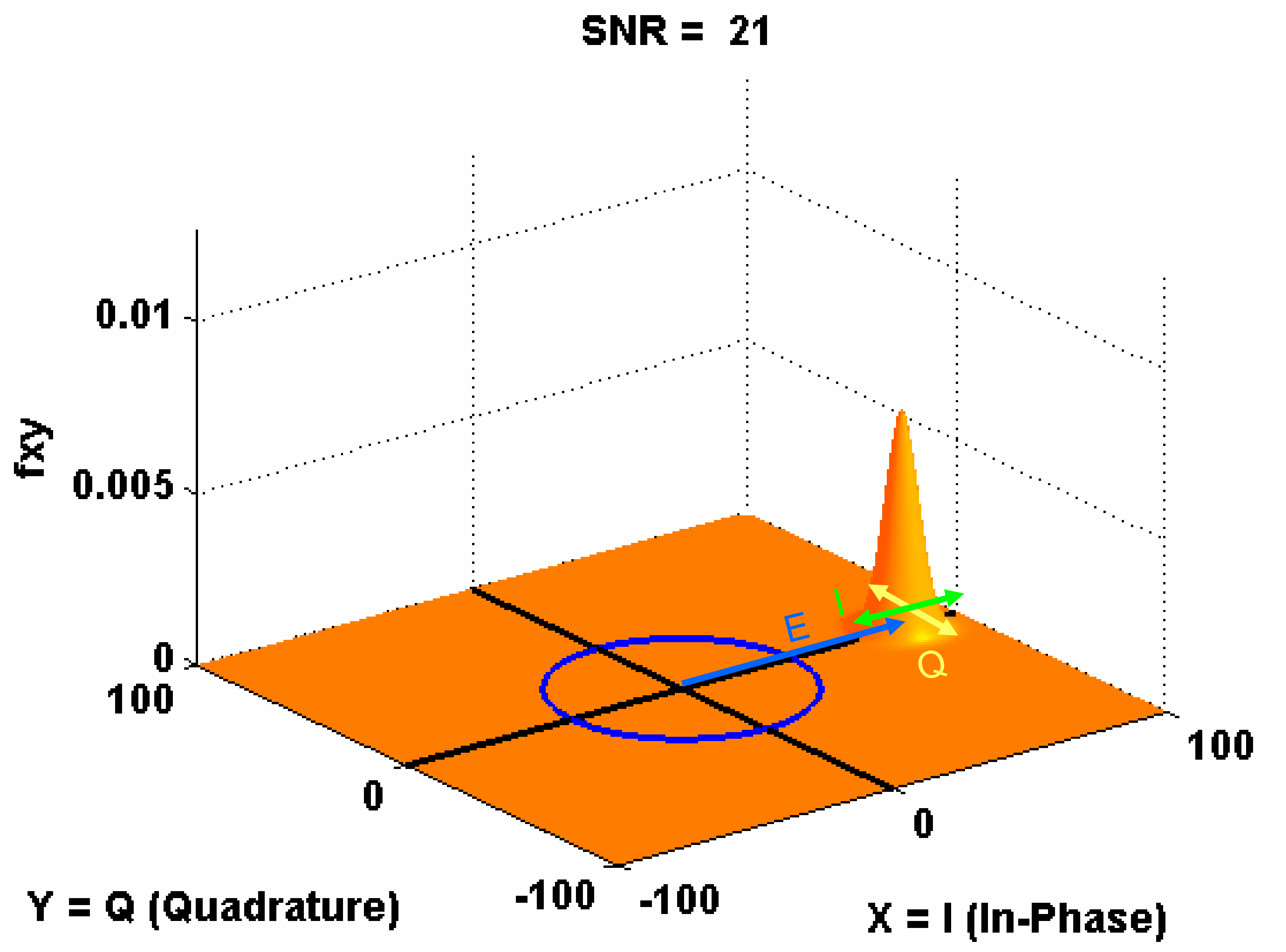
$$P\{r \leq Z \leq r+dr\} = \iint_{A=\pi r^2} f_{xy}(x,y)\,dy\,dx$$

$$P\{r \leq Z \leq r+dr\} = \iint_{A=\pi r^2} \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_y^2}}\,dy\,dx$$

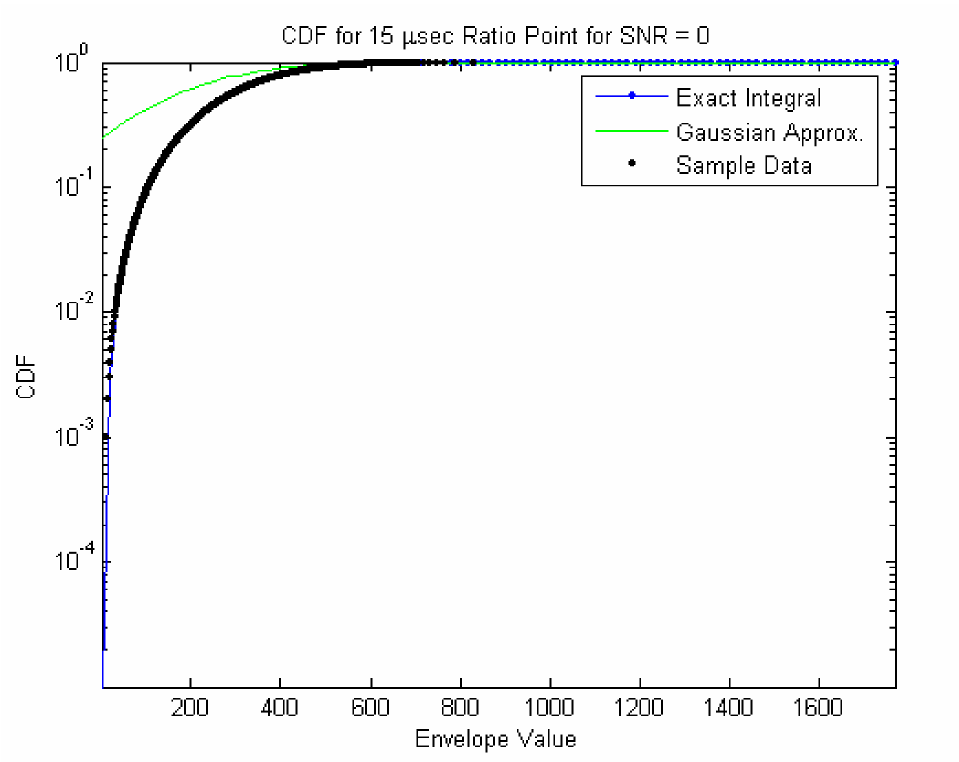
$$= \int_{x=-r}^{x=r} \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \int_{y=-\sqrt{r^2-x^2}}^{y=\sqrt{r^2-x^2}} \frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_y^2}}\,dy\,dx$$

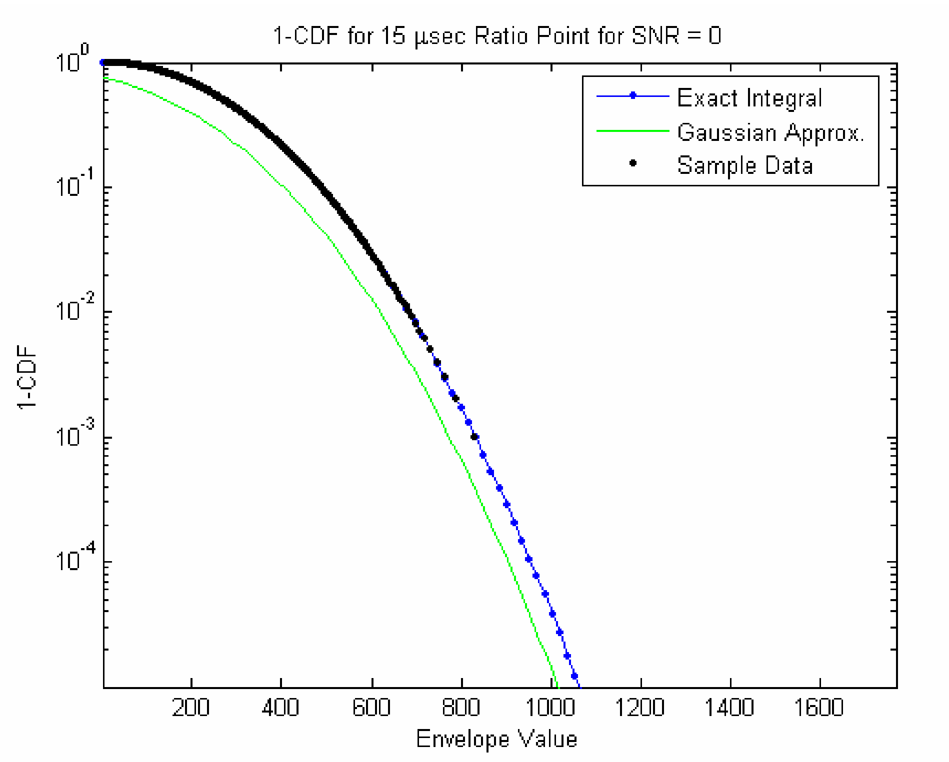

SNR = -Inf
fxy
0.01
0.005
0
Constant Envelope
100
0
-100
Y = Q (Quadrature)
-100
0
100
X = I (In-Phase)


SNR = 15
fxy
0.01
0.005
0
100
0
-100
-100
0
100
Y = Q (Quadrature)
X = I (In-Phase)

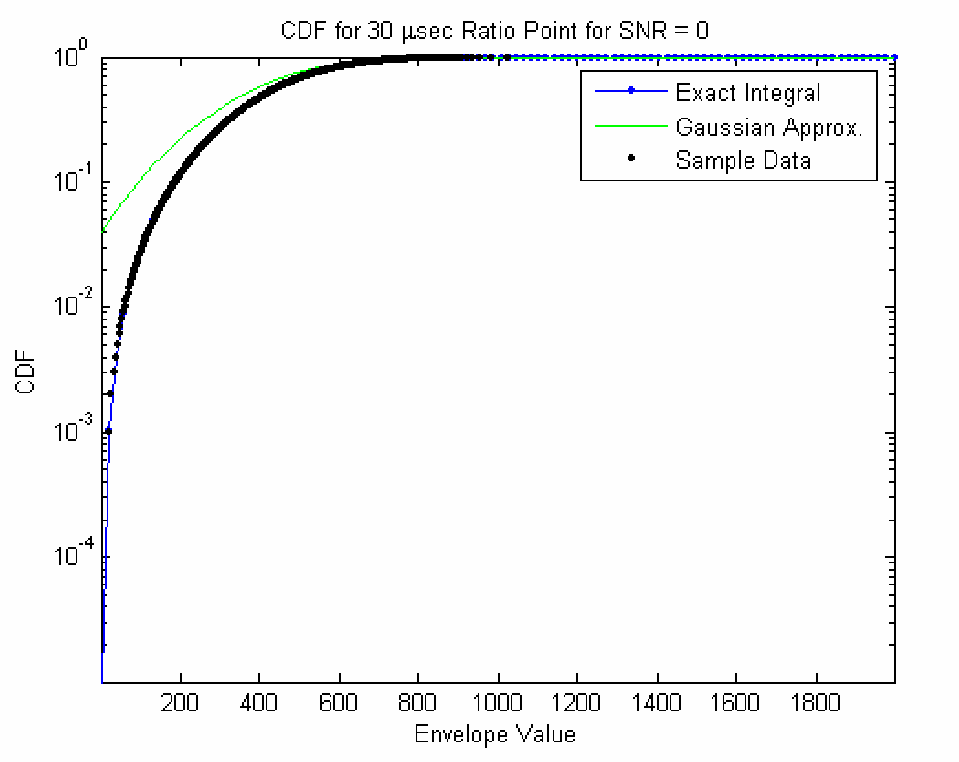

SNR = 21
fxy
0.01
0.005
0
100
0
-100
-100
0
100
E
I
Q
Y = Q (Quadrature)
X = I (In-Phase)

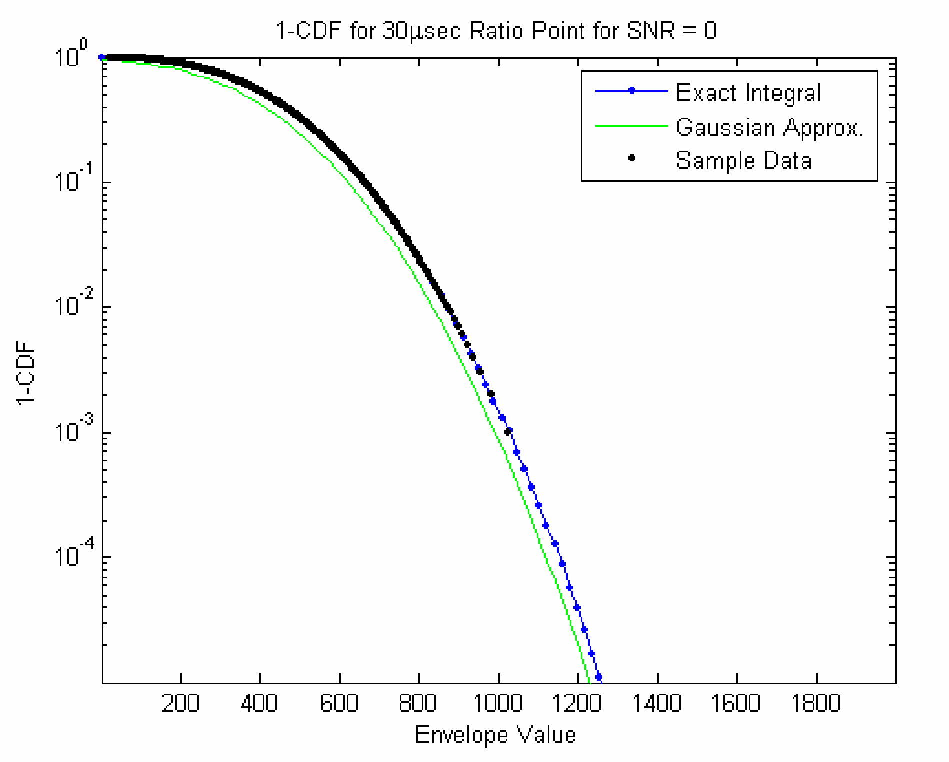
15us Envelope Statistics (Low SNR)



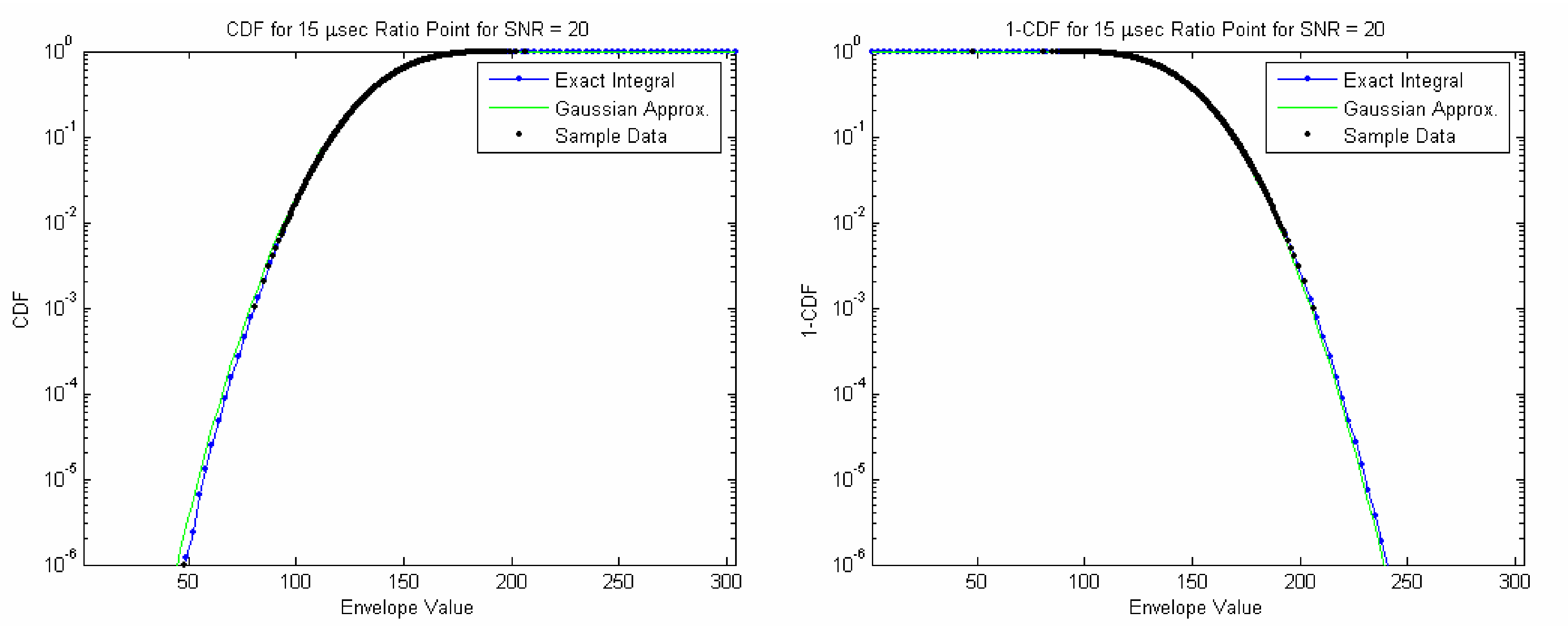


30us Envelope Statistics





15us Envelope Statistics (High SNR)



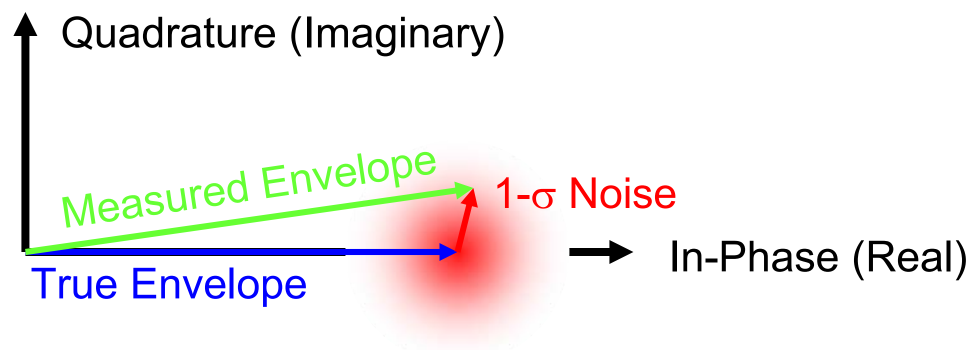
Envelope Conclusions

- For small SNR, Envelope values are related to the RSS of two Gaussian variables (Rayleigh)
- As SNR gets larger, Envelope values becomes dependent on only one variable (or a linear combination of the two) and therefore more Gaussian.
  - Tracking with high confidence ~ large SNRs

Ratio Statistics

P[Wrong Cycle] Gaussian Noise

- Given two ideal envelope values, add Gaussian noise inversely proportional to SNR



- Take ratio of two envelope values, 15 usec apart (e.g. 15 usec / 30 usec)

$$\text{Ratio}_{30usec} = \frac{\text{Meas. Envelope 15usec}}{\text{Meas. Envelope 30usec}}$$

- Compare to ratio of the envelope that is shifted by +/- 5 usec

$$\text{Ratio}_{25usec} < \text{Ratio}_{30usec} < \text{Ratio}_{35usec}$$

- Cycle slip if my ratio falls outside of range

PDF of Ratios

- Any ratio of two random variables where q = y/x

$$f_Q(q) = \int_{x=0}^{\infty} x\, f_X(x)\, f_Y(qx)\, dx$$

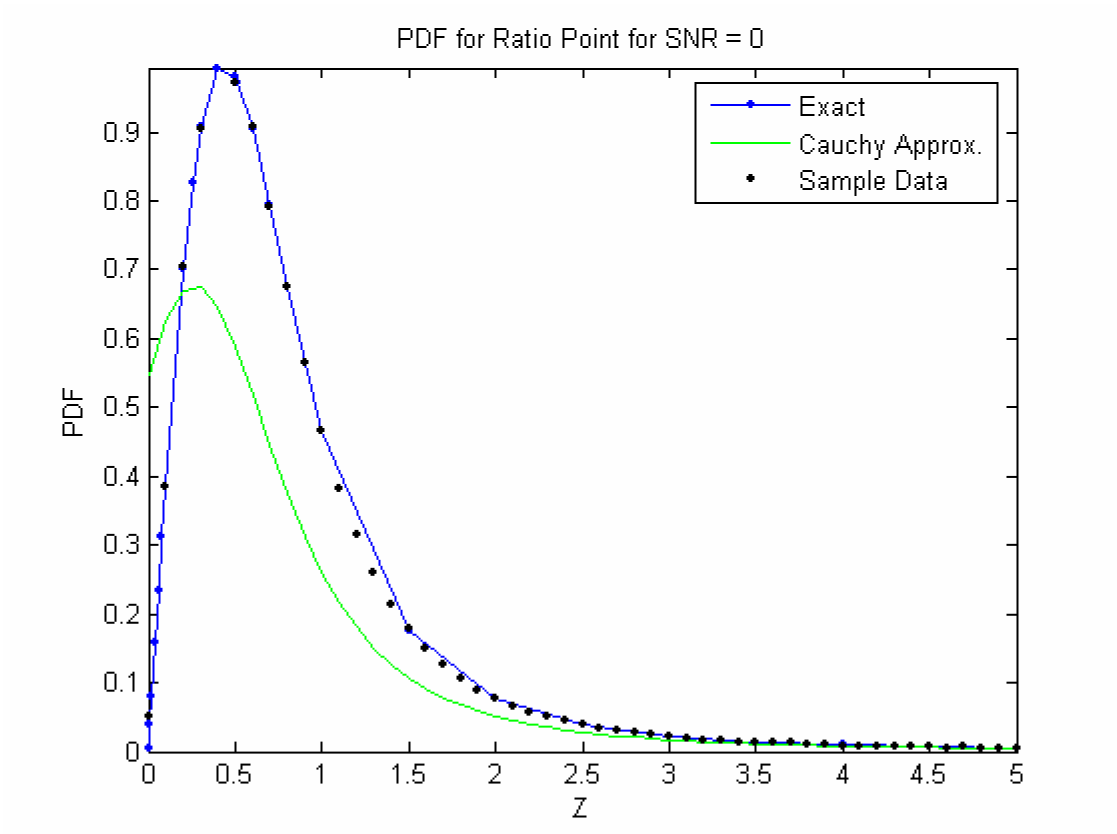
- If x & y are ~N(0,1)

$$f_{Cauchy}(t) = \frac{1}{\pi(1+t^2)}$$

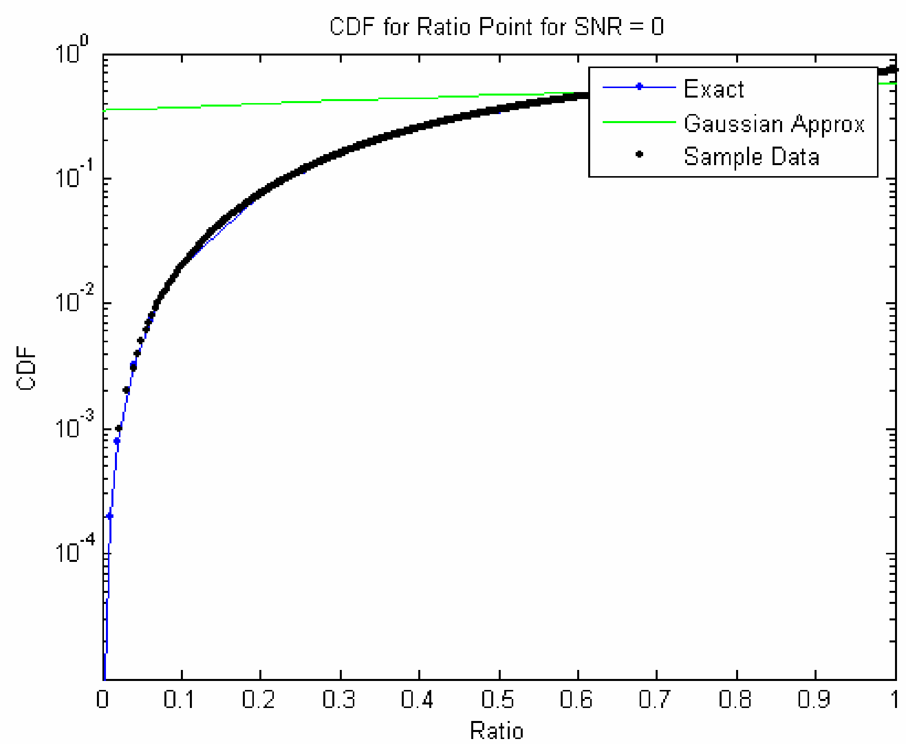
- If x & y are Gaussian with vanishing probabilities at zero

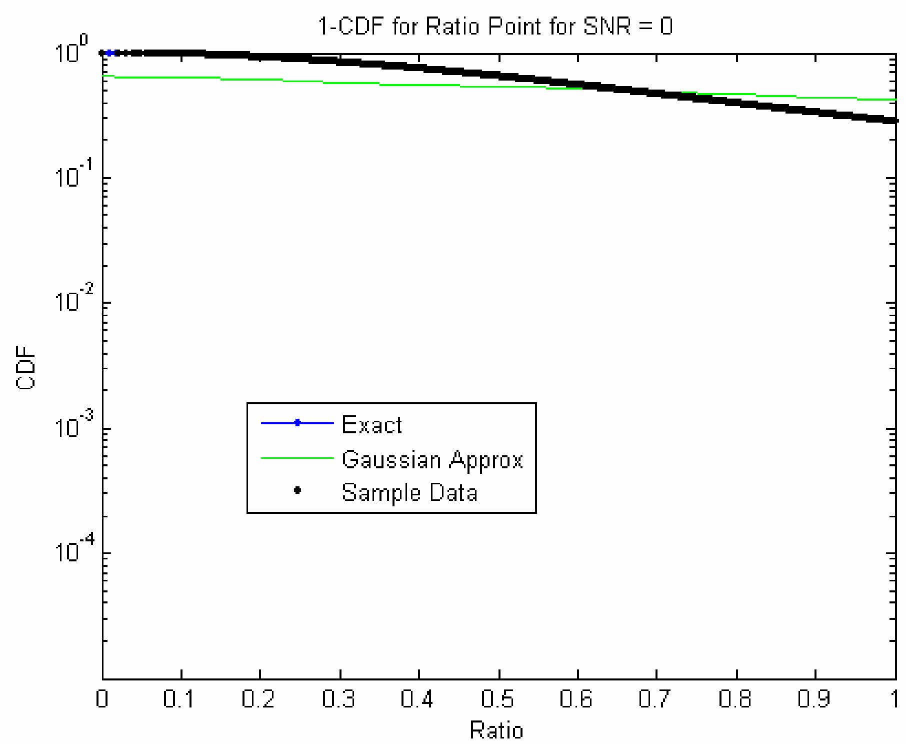
$$f_C(t) = \frac{\sigma_A^2 \mu_B + \sigma_B^2 \mu_A t}{\sqrt{2\pi}\,(\sigma_A^2 + \sigma_B^2 t^2)^{3/2}} \exp\left[-\frac{(\mu_A - \mu_B t)^2}{(\sigma_A^2 + \sigma_B^2 t^2)} \right]$$

PDF Ratio (Low SNR)

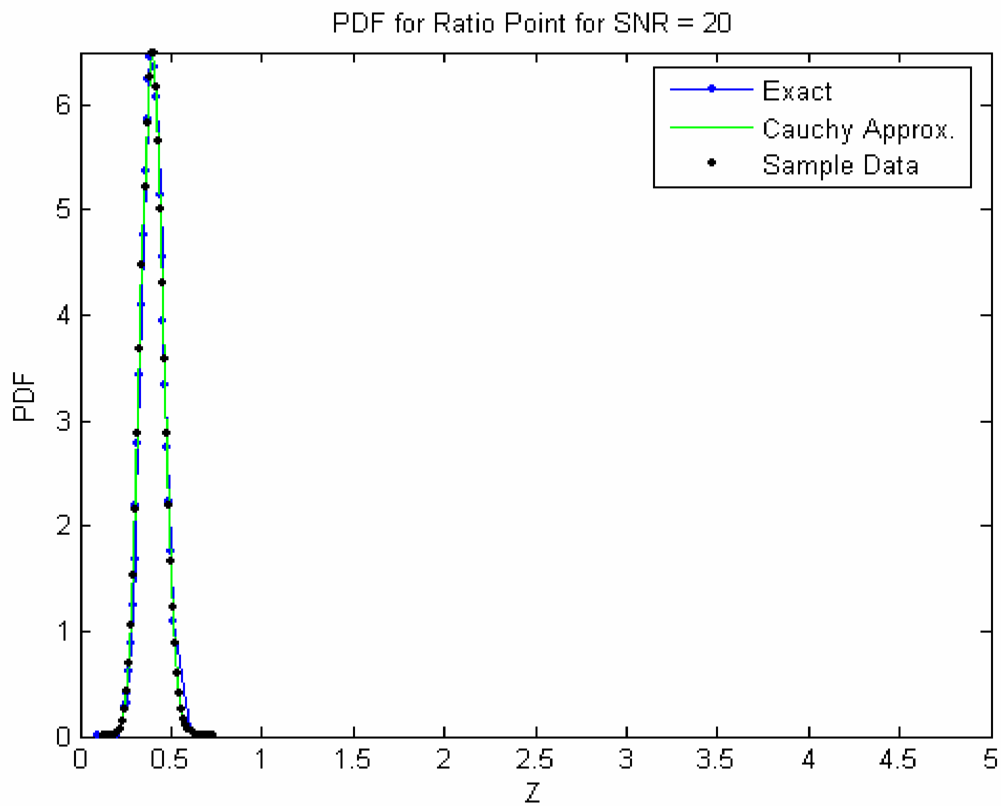


Ratio CDFs (Low SNR)

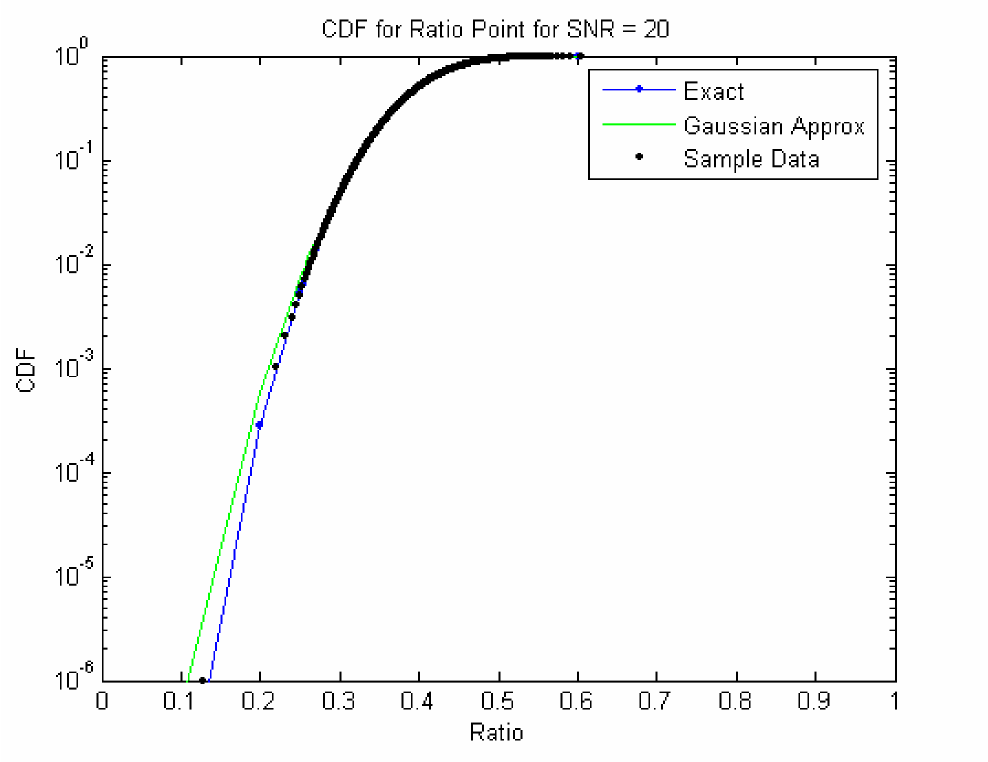


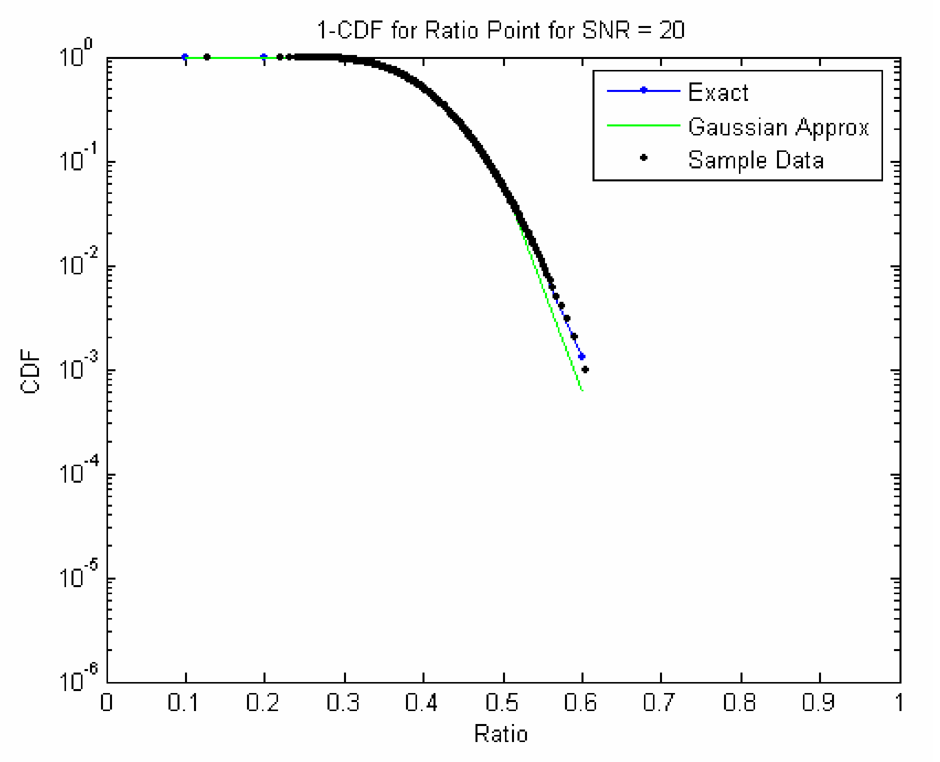


PDF of Ratio


PDF for Ratio Point for SNR = 20
Exact
Cauchy Approx.
Sample Data
PDF
Z
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

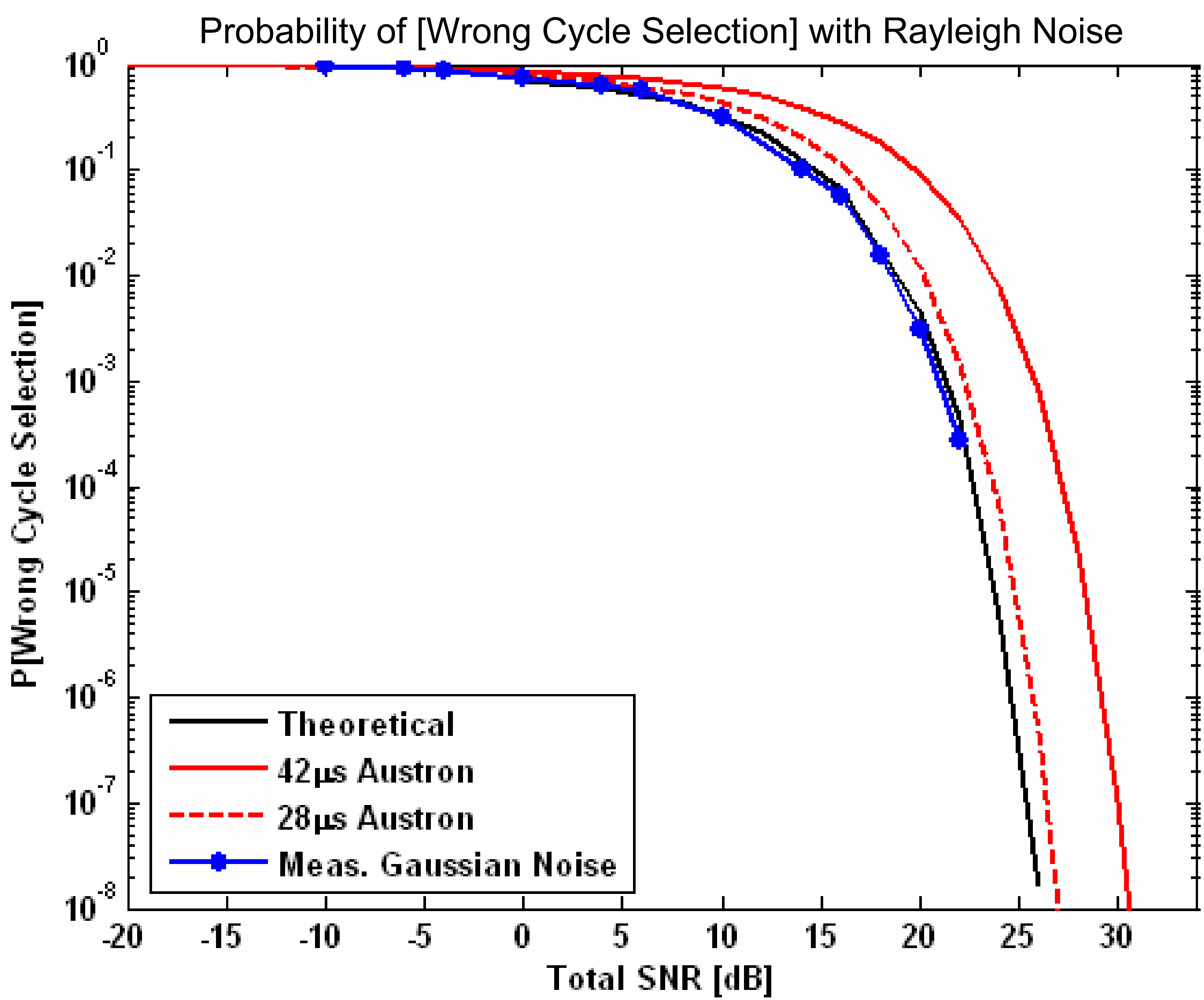
CDF of Ratio (High SNR)





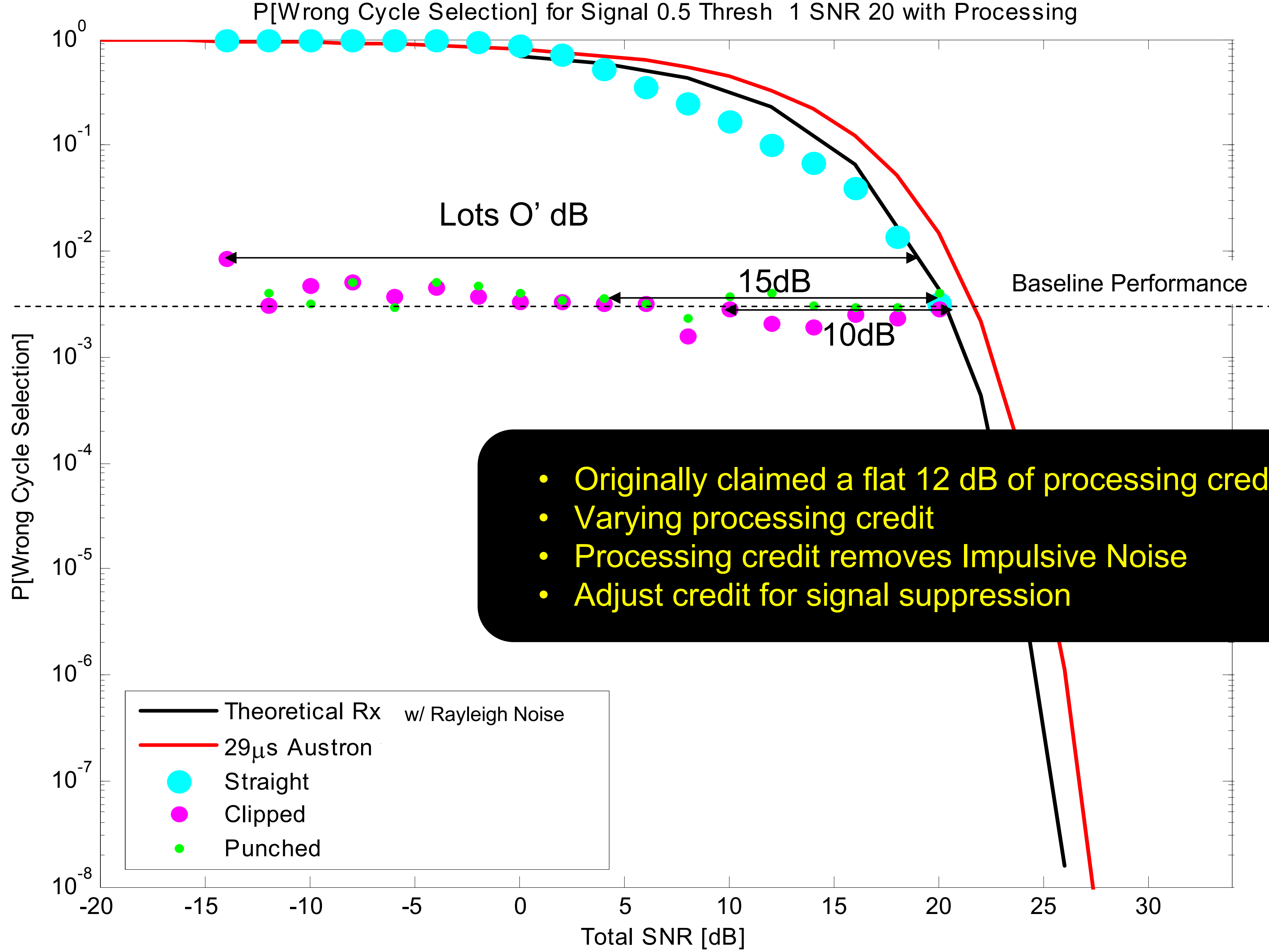
P[Wrong Cycle] Gaussian Noise

- Since the Envelope is Gaussian the ratio of them approaches Cauchy as SNR gets large.
- Old Austron 5000 estimate
  - P[Wrong Cycle] ~ 42 us /sqrt(N * SNR)
- Estimate for Modern Loran receiver
  - P[Wrong Cycle] ~ 29 us /sqrt(N * SNR)



Probability of [Wrong Cycle Selection] with Rayleigh Noise
P[Wrong Cycle Selection]
Total SNR [dB]
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
-20
-15
-10
-5
0
5
10
15
20
25
30
Theoretical
42μs Austron
28μs Austron
Meas. Gaussian Noise

Estimated Probability of [Wrong Cycle Selection]



- Originally claimed a flat 12 dB of processing credit
- Varying processing credit
- Processing credit removes Impulsive Noise
- Adjust credit for signal suppression

Summary & Future Work

- Formed an exact representation of P[wrong cycle] under Gaussian noise
- Probability of correct cycle is ~ SNR
- Use the Gaussian noise case as our reference
  - Compare our receiver performance in the presence of non-Gaussian noise
  - Show the effects of non-linear processing
- Showed the performance of non-linear processing on simulated Loran
- Working on 2006 data with real Loran signals
- Possible extension using ITU predicted noise pdfs